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Damping of Coupled-Bunch Growth by Self-Excited Cavity

K.-Y. Ng
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

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1 INTRODUCTION

The bunch area of a typical bunch in the Fermilab Booster across transition had been measured by Crisp[1] and is shown in Fig. 1. We see that the bunch area is roughly constant before transition but increases abruptly after transition. The mountain-range plot of Fig. 2a reveals quadrupole oscillations of the bunch after transition. This is due to space-charge effects which alter the rf potential and allow for a shorter bunch after transition. Thus, after transition, the bunch does not fit the bucket and starts tumbling.[2] Figure 2b shows the same mountain-range plot but with a transition jump mechanism installed. With the elimination of space-charge effects, we begin to see a coupled-bunch pattern. Several suggestions have been proposed to cure the coupled-bunch coherent growth. These include the damping of the particular resonances that are driving the growth, a fast damper to decouple the coherency, a Landau cavity to provide more Landau damping, and a self-exciting cavity to cancel the coupling.[3] In this note the possibility of the self-exciting cavity is examined.

2 THEORY

For the rigid dipole mode and coupled-bunch mode s, the coherent shift in synchrotron tune without any Landau damping is given by[4]

$$\Delta\nu_s = \frac{i\eta I_b k}{4\pi\beta^2\nu_{s0}(E/e)} Z_{eff} \qquad s = 0, 1, 2, \ldots, k-1 \ , \tag{1}$$

with the effective impedance defined as

$$Z_{eff} = \sum_{p=-\infty}^{\infty} \nu_p e^{-(\nu_p\sigma_\phi)^2} Z(\nu_p\omega_0) \ , \tag{2}$$

and

$$\nu_p = pk + s + \nu_{s0} \ . \tag{3}$$

In the above, η is the frequency flip parameter, I_b is the average current of one bunch, ν_{s0} is the synchrotron tune at zero amplitude, E/e the total energy of a bunch particle per unit charge, β is the particle velocity divided by the velocity of light, $\omega_0/2\pi$ is the frequency of revolution of the particle around the Booster Ring, and $k = 84$ is the number of identical bunches each with RMS length σ_ϕ in radians. The longitudinal impedance $Z(\nu_p\omega_0)$ consists of a number of sharp resonances of the rf cavities.

Take for example the resonance at $f_r = 85.8$ MHz, shunt impedance $Z_s = 1.564$ MΩ, and quality factor $Q = 3378$. As the particles accelerate, a spectral line of the bunch corresponding to coupled-bunch mode $s = 53$ crosses the resonance at positive frequency while a line corresponding to $s = 31$ crosses the resonance

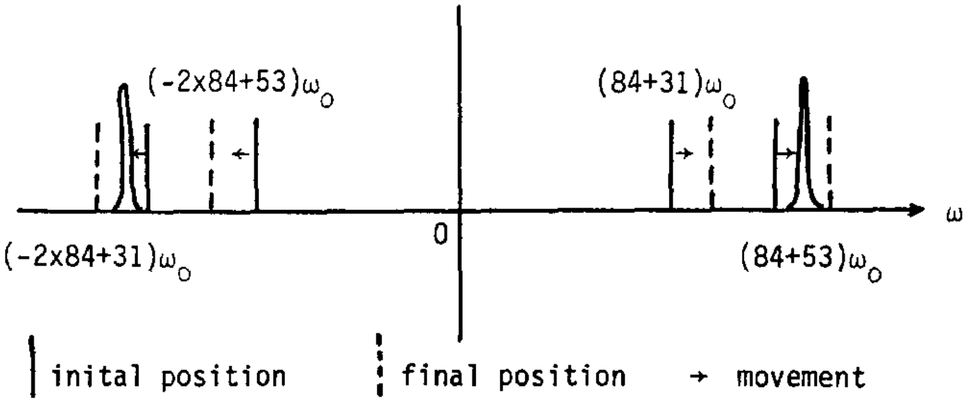


Figure 3

at negative frequency at a slightly later time (at exactly the same time if $\nu_{s0} = 0$). According to Eq. (1), mode $s = 53$ will grow while mode $s = 31$ will be damped (see Fig. 3).

Griffin[3] pointed out that if a complementary resonance is placed near $(84 + 31)\omega_0$, a $s = 53$ line will cross this resonance at negative frequency giving a damping of the $s = 53$ mode. . The $s = 31$ will grow, however, because the $(84 + 31)\omega_0$ line will cross the same resonance at positive frequency. Hopefully, this growth may be cancelled by the damping supplied by the original resonance. This additional resonance can be furnished by the excitation of a carefully-tuned cavity by the bunch. The merit of such a device lies in its simplicity.

Let the angular frequencies of the original and complementary resonances be ω_r and ω_r' respectively. When a spectral line corresponding to coupled-bunch mode s crosses the original resonance, suppose that the angular revolution frequency is ω_0. We can write:

$$\omega_r = (n_p + \nu_{s0})\omega_0 \ , \tag{4}$$

where

$$\nu_p = n_p + \nu_{s0} \ , \tag{5}$$

and

$$n_p = pk + s \qquad p = \text{some integer} \ . \tag{6}$$

Note that in order for this resonance to drive the growth of mode s, ν_p must be positive. If we want another spectral line of mode s to cross the complementary resonance at the same time and receive a damping drive, we must have

$$-\omega_r' = (-n_{p'} + \nu_{s0})\omega_0 \ , \tag{7}$$

with

$$-n_{p'} = -p'k + s \ , \tag{8}$$

for some integer p'. Note that

$$\nu_{p'} = -n_{p'} + \nu_{s0} \tag{9}$$

must be negative to effect a damping. Combining Eqs. (1) and (4), we get

$$\omega_r' = \omega_r \frac{n_{p'} - \nu_{s0}}{n_p + \nu_{s0}} \ . \tag{10}$$

On the other hand, the original resonance will cause damping and the complementary resonance will cause growth for coupled-bunch mode $k - s$. One of the $k - s$ spectral lines, however, will cross the original resonance at a slightly later time after the mode s line. Suppose that the angular revolution frequency at crossing the original is ω_0' and we want the other $k - s$ line to cross the complementary resonance at the same time. We have

$$-\omega_r = (-n_p + \nu_{s0})\omega' \ , \tag{11}$$

and

$$\omega_r' = (n_{p'} + \nu_{s0})\omega' \ , \tag{12}$$

where n_p and $n_{p'}$ are given by Eqs. (6) and (9) respectively. Note that $-n_p + \nu_{s0}$ is negative and $n_{p'} + \nu_{s0}$ is positive. We therefore get

$$\omega_r' = \omega_r \frac{n_{p'} + \nu_{s0}}{n_p - \nu_{s0}} \ . \tag{13}$$

We see that Eqs. (10) and (13) are incompatible. However, since ν_{s0} is extremely small, we can choose the position of the complementary resonance at

$$\omega_r' = \omega_r \frac{n_{p'}}{n_p} \ , \tag{14}$$

which is a shade too big for mode s and a shade too small for mode $k - s$. This is nice, however, because for both modes damping will come slightly after the growth.

Next, we want to compute the time Δt for a spectral line to cross a resonance of width $\Delta\omega_r$ and centered at ω_r. Near ω_r, the angular velocity varies according to

$$\omega_0(t) = \alpha t \ , \tag{15}$$

where α is a constant and $t = 0$ is the time when the resonance is crossed. We have then

$$\Delta\omega_r = \nu_p \alpha \Delta t \ , \tag{16}$$

where ν_p is given by Eq. (5). Recalling that

$$\Delta\omega_r = \nu_p \omega_0 / Q \ , \tag{17}$$

we arrive at

$$\Delta t = \frac{1}{\alpha Q} \ . \tag{18}$$

To obtain a complete damping of the growth, from Eq. (1), the shunt impedance Z_s' and the quality factor Q' are related to the original shunt impedance Z_s and the original quality factor Q by

$$|\nu_{p'}|e^{-(\nu_{p'}\sigma_\phi)^2} Z_s'/Q'^2 = |\nu_p|e^{-(\nu_p\sigma_\phi)^2} Z_s/Q^2 \ . \tag{19}$$

The extra factors of Q and Q, come from the crossing time Δt in Eq. (18). In most cases, the exponents in Eq. (19) are rather small; the exponentials can therefore be considered as unity and Eq. (19) simplifies to

$$\frac{\nu_{p'} Z_s'}{Q'^2} = \frac{\nu_p Z_s}{Q^2} \ . \tag{20}$$

If we further demand that the time periods for growth and damping (or the crossing times of the two resonance) are exactly the same, Eq. (18) leads to

$$Q' = Q \ , \tag{21}$$

and Eq. (20) becomes

$$Z_s' = Z_s \frac{|\nu_p|}{|\nu_{p'}|} \ . \tag{22}$$

In reality, we do not know the shunt impedance Z_s and the quality factor Q this accurately. Fortunately, we always have a small amount of Landau damping as a result of the synchrotron frequency spread. Thus, the above criteria (21) and (22) need not be strictly satisfied and the shunt impedance of the complementary resonance can be fine-tuned to give the best damping result.

3 RESULTS

We try to add a complementary resonance to the resonance at 85.80 MHz with the following parameters:

	f_r (MHz)	Z_s (MΩ)	Q
original	85.80	1.564	3378
complementary	72.02	1.887	3378

according to Eqs. (21) and(22). The growth rates as functions of energy for modes 53 and 31 are shown in Figs. 4 and 5. We see that in each case the damping comes in slightly after the growth as expected. Also in each case, the damping does not cancel the growth completely. However, the residual growths are damped by Landau damping. The growth rates (or damping rates) for all the $k = 84$ modes are tabulated in Table I for each resonance. The last column shows the total growth rates with a Landau damping included. The amount of Landau damping is taken as[4]

$$\Delta\omega = -0.097\nu_{s0}\omega_0\sigma_\phi^2 \, . \tag{23}$$

4 MANY MODES

Table I shows that the resonance at 85.80 MHz is crossed mainly by mode 53 only (or mode 31 for damping). Although there are many neighboring modes that also grow, those growth rates are rather small because only the tail of the resonance is crossed. Under this situation, we have shown in the above sections that a complementary resonance and a small Landau damping can fix all the modes. In the Fermilab Booster, there is another annoying resonance at $\omega_r/2\pi = 167.2$ MHz, shunt impedance $Z_s = 0.0756$ MΩ, and quality factor $Q = 1959$. This resonance peak, being at a higher frequency, is crossed by spectral lines of modes 16, 15, 14 as the bunches are accelerated from transition to 8.9 GeV. As shown in Fig. 6, mode 16 is excited first and then mode 15 and lastly mode 14. To cancel mode 16 at the time it is growing or just slightly after it grows, we need to put a complementary resonance at ω_r' (and of course $-\omega_r'$) between spectral lines 15 and 16 of another band. This resonance, however, will excite a growth of mode 14 immediately, but the original resonance will only provide the necessary damping at a much later time. Also, this complementary resonance will be crossed by a spectral line of mode 71 and possibly mode 72, and no damping will be provided for these modes by the original resonance at all. We therefore conclude that when the peak of a resonance is crossed by spectral lines of more than one mode, the addition of a complementary resonance will not cure the growths, because although it will damp a certain mode it will cause growths in other modes that cannot be damped by the original resonance.

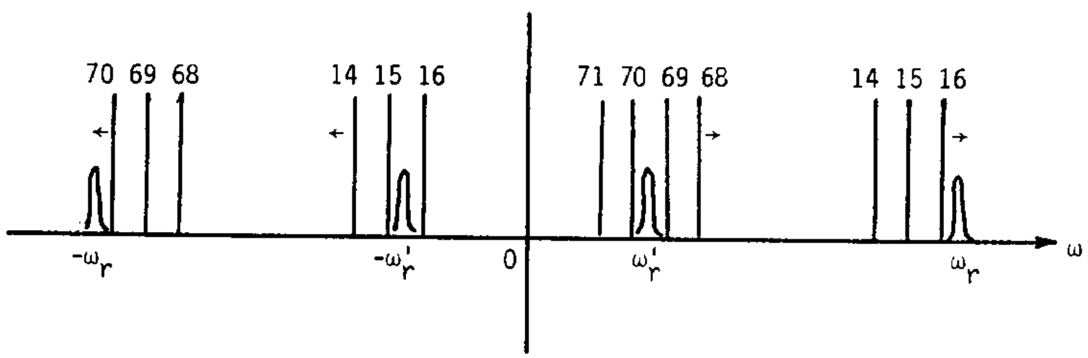


Figure 6

REFERENCES

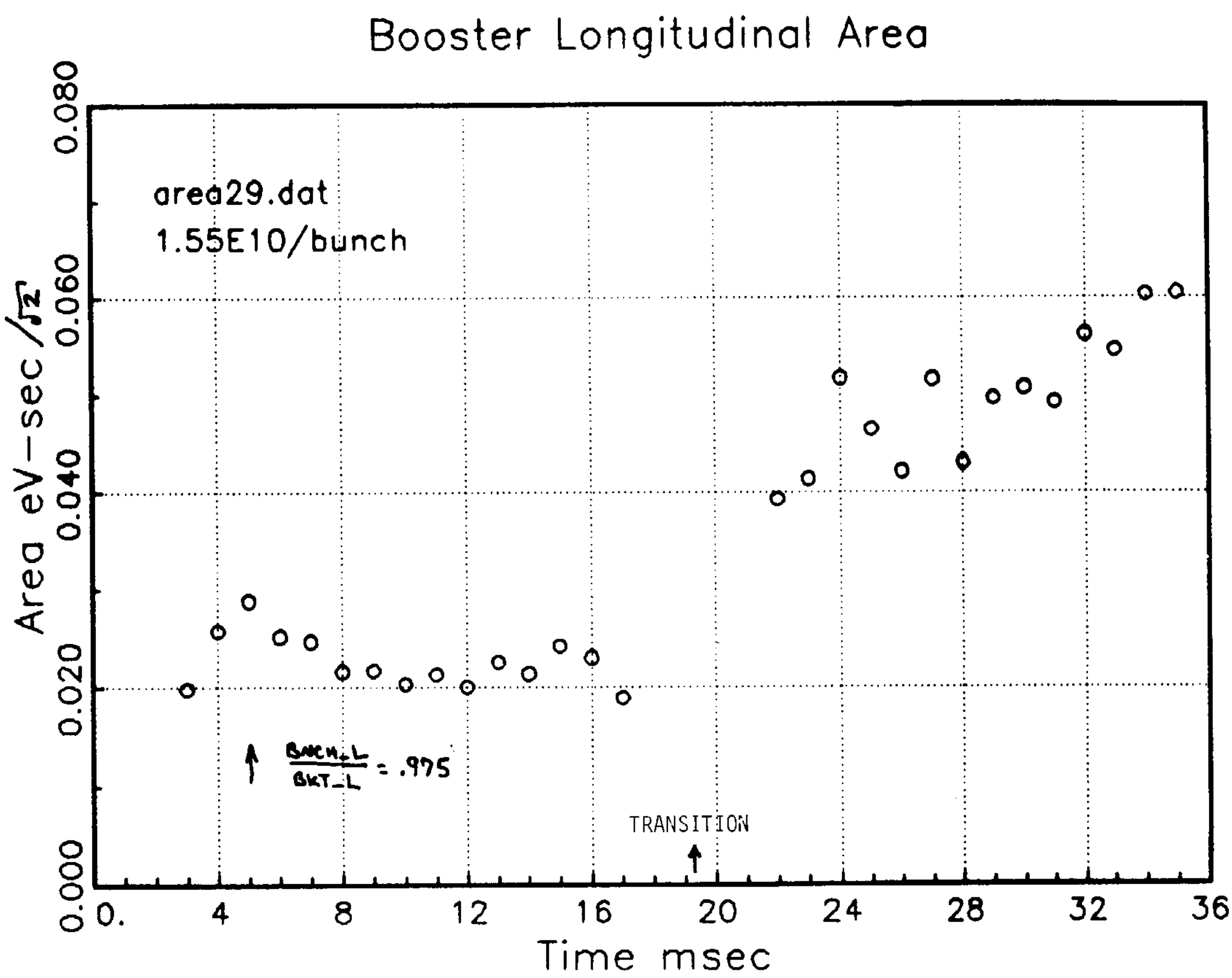
1. J. Crisp, private communication, 1985.
2. J. Crisp, private communication, 1987.
3. J. Griffin, private communication, 1987.
4. J.M. Wang, BNL Report BNL-51302, 1980.

Table I

2------ Original resonance
3------ Complementary resonance
L.DAMP- Landau damping included

INTERNAL BUNCH MODE = 1

COUPLED MODE	INTEGRATED GROWTH WITH FOLLOWING RESONANCES: 2	3	TOTAL	L.DAMP
0	0.0000	0.0000	0.0000	-0.5969
1	0.0000	0.0000	0.0000	-0.5970
2	0.0000	0.0000	0.0000	-0.5970
3	0.0000	0.0000	0.0000	-0.5970
4	0.0000	0.0000	0.0000	-0.5970
5	-0.0001	0.0000	0.0000	-0.5970
6	-0.0001	0.0000	0.0000	-0.5970
7	-0.0001	0.0000	-0.0001	-0.5970
8	-0.0001	0.0000	-0.0001	-0.5970
9	-0.0001	0.0000	-0.0001	-0.5970
10	-0.0001	0.0000	-0.0001	-0.5970
11	-0.0001	0.0000	-0.0001	-0.5970
12	-0.0002	0.0001	-0.0001	-0.5971
13	-0.0002	0.0001	-0.0001	-0.5971
14	-0.0002	0.0001	-0.0001	-0.5971
15	-0.0003	0.0001	-0.0002	-0.5971
16	-0.0003	0.0001	-0.0002	-0.5971
17	-0.0003	0.0002	-0.0002	-0.5971
18	-0.0004	0.0002	-0.0002	-0.5972
19	-0.0005	0.0002	-0.0002	-0.5972
20	-0.0006	0.0003	-0.0003	-0.5972
21	-0.0007	0.0004	-0.0003	-0.5973
22	-0.0009	0.0005	-0.0004	-0.5973
23	-0.0011	0.0006	-0.0004	-0.5974
24	-0.0014	0.0009	-0.0005	-0.5975
25	-0.0019	0.0013	-0.0007	-0.5976
26	-0.0027	0.0019	-0.0008	-0.5978
27	-0.0042	0.0031	-0.0011	-0.5980
28	-0.0074	0.0060	-0.0014	-0.5983
29	-0.0166	0.0152	-0.0014	-0.5984
30	-0.0984	0.0906	-0.0079	-0.6048
31	-6.3067	6.6813	0.3746	-0.2223
32	-0.1486	0.0552	-0.0934	-0.6904
33	-0.0220	0.0119	-0.0102	-0.6071
34	-0.0085	0.0053	-0.0032	-0.6002
35	-0.0044	0.0030	-0.0014	-0.5984
36	-0.0026	0.0019	-0.0008	-0.5977
37	-0.0017	0.0012	-0.0004	-0.5974
38	-0.0011	0.0008	-0.0003	-0.5972
39	-0.0007	0.0006	-0.0002	-0.5971
40	-0.0004	0.0003	-0.0001	-0.5970
41	-0.0002	0.0002	0.0000	-0.5970
42	0.0000	0.0000	0.0000	-0.5969
43	0.0002	-0.0002	0.0000	-0.5969
44	0.0004	-0.0003	0.0001	-0.5969
45	0.0007	-0.0006	0.0002	-0.5968
46	0.0011	-0.0008	0.0003	-0.5967
47	0.0017	-0.0012	0.0004	-0.5965
48	0.0026	-0.0019	0.0008	-0.5962
49	0.0044	-0.0030	0.0014	-0.5955
50	0.0086	-0.0053	0.0032	-0.5937
51	0.0222	-0.0120	0.0103	-0.5867
52	0.1524	-0.0561	0.0963	-0.5006
53	6.4805	-6.4796	0.0009	-0.5960
54	0.0962	-0.0886	0.0076	-0.5893
55	0.0165	-0.0150	0.0014	-0.5955
56	0.0074	-0.0060	0.0014	-0.5956
57	0.0042	-0.0031	0.0011	-0.5959
58	0.0027	-0.0019	0.0008	-0.5961
59	0.0019	-0.0013	0.0007	-0.5963
60	0.0014	-0.0009	0.0005	-0.5964
61	0.0011	-0.0006	0.0004	-0.5965
62	0.0009	-0.0005	0.0004	-0.5966
63	0.0007	-0.0004	0.0003	-0.5966
64	0.0006	-0.0003	0.0003	-0.5967
65	0.0005	-0.0002	0.0002	-0.5967
66	0.0004	-0.0002	0.0002	-0.5967
67	0.0003	-0.0002	0.0002	-0.5968
68	0.0003	-0.0001	0.0002	-0.5968
69	0.0003	-0.0001	0.0002	-0.5968
70	0.0002	-0.0001	0.0001	-0.5968
71	0.0002	-0.0001	0.0001	-0.5968
72	0.0002	-0.0001	0.0001	-0.5968
73	0.0001	0.0000	0.0001	-0.5969
74	0.0001	0.0000	0.0001	-0.5969
75	0.0001	0.0000	0.0001	-0.5969
76	0.0001	0.0000	0.0001	-0.5969
77	0.0001	0.0000	0.0001	-0.5969
78	0.0001	0.0000	0.0000	-0.5969
79	0.0001	0.0000	0.0000	-0.5969
80	0.0000	0.0000	0.0000	-0.5969
81	0.0000	0.0000	0.0000	-0.5969
82	0.0000	0.0000	0.0000	-0.5969
83	0.0000	0.0000	0.0000	-0.5969



Booster Longitudinal Area
area29.dat
1.55E10/bunch
BNCH_L / BKT_L = .975
TRANSITION
Area eV-sec/√2
0.000
0.020
0.040
0.060
0.080
0.
4
8
12
16
20
24
28
32
36
Time msec

Figure 1

LEFT COLUMN — δ_I ON

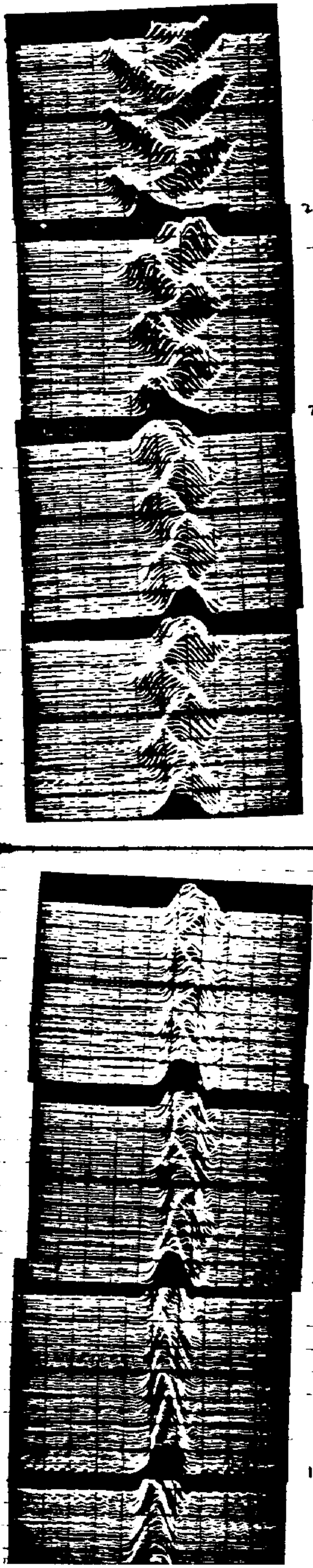
RT COLUMN — δ_I OFF

δ_I 18630

IRT 18660 — IRT 17500

$1.5\,10^{11}$ /H

all pictures
10 turns/trace
63 sweeps



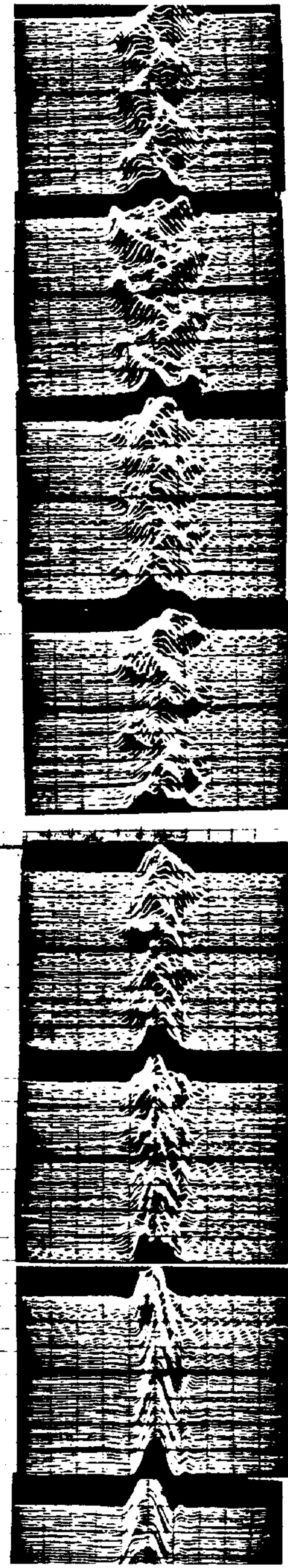


Fig 2

(b) (a)

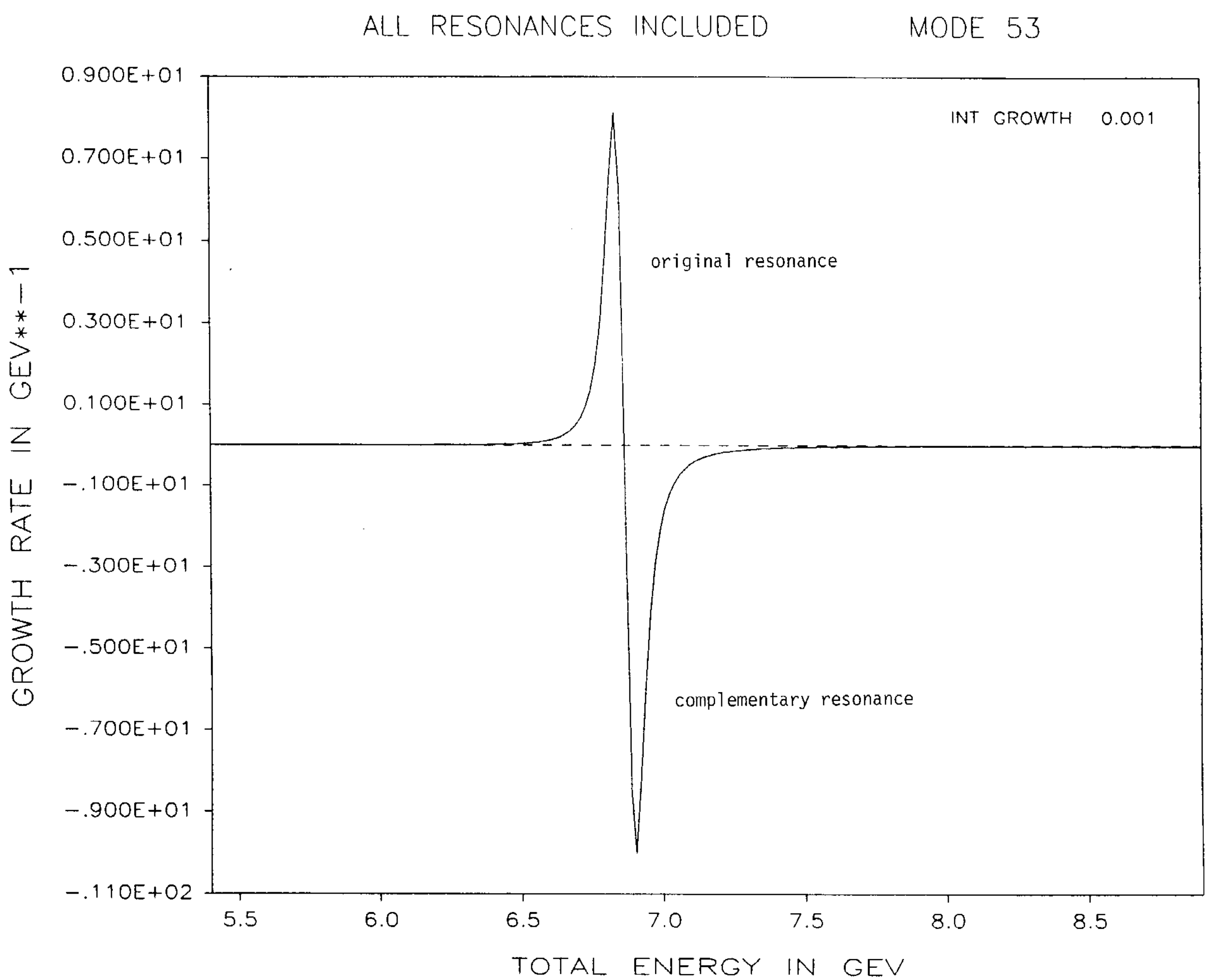

ALL RESONANCES INCLUDED
MODE 53
INT GROWTH 0.001
original resonance
complementary resonance
0.900E+01
0.700E+01
0.500E+01
0.300E+01
0.100E+01
-.100E+01
-.300E+01
-.500E+01
-.700E+01
-.900E+01
-.110E+02
5.5
6.0
6.5
7.0
7.5
8.0
8.5
GROWTH RATE IN GEV**-1
TOTAL ENERGY IN GEV

Figure 4

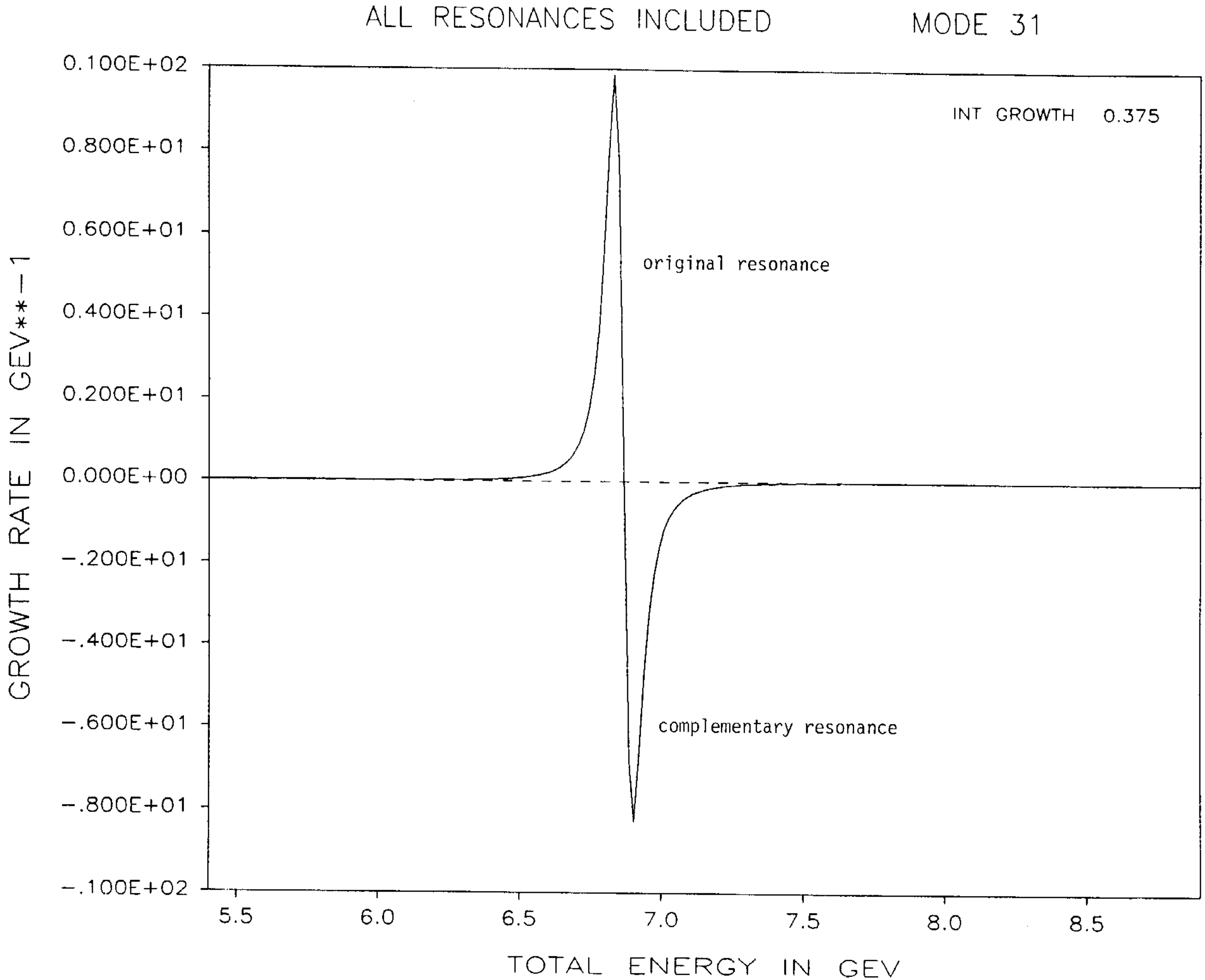

ALL RESONANCES INCLUDED
MODE 31
INT GROWTH 0.375
original resonance
complementary resonance
0.100E+02
0.800E+01
0.600E+01
0.400E+01
0.200E+01
0.000E+00
-.200E+01
-.400E+01
-.600E+01
-.800E+01
-.100E+02
GROWTH RATE IN GEV**-1
5.5
6.0
6.5
7.0
7.5
8.0
8.5
TOTAL ENERGY IN GEV

Figure 5